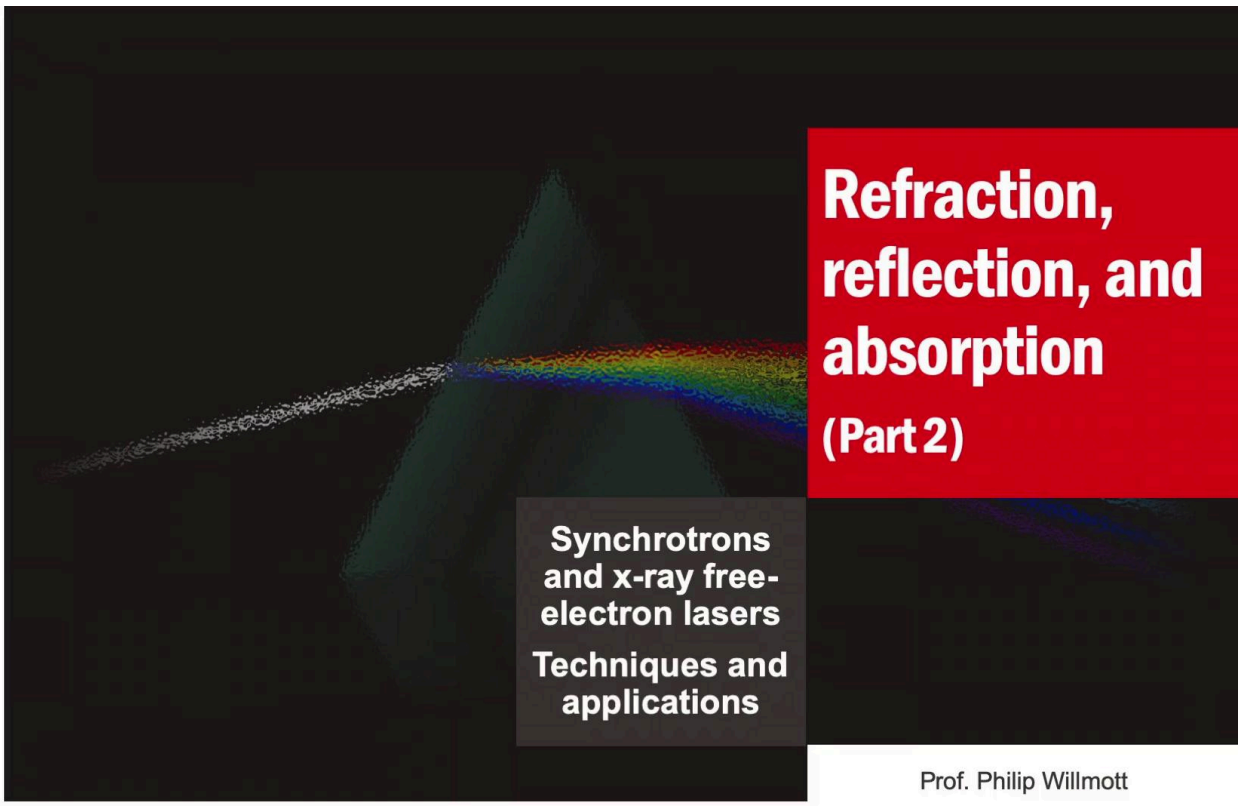

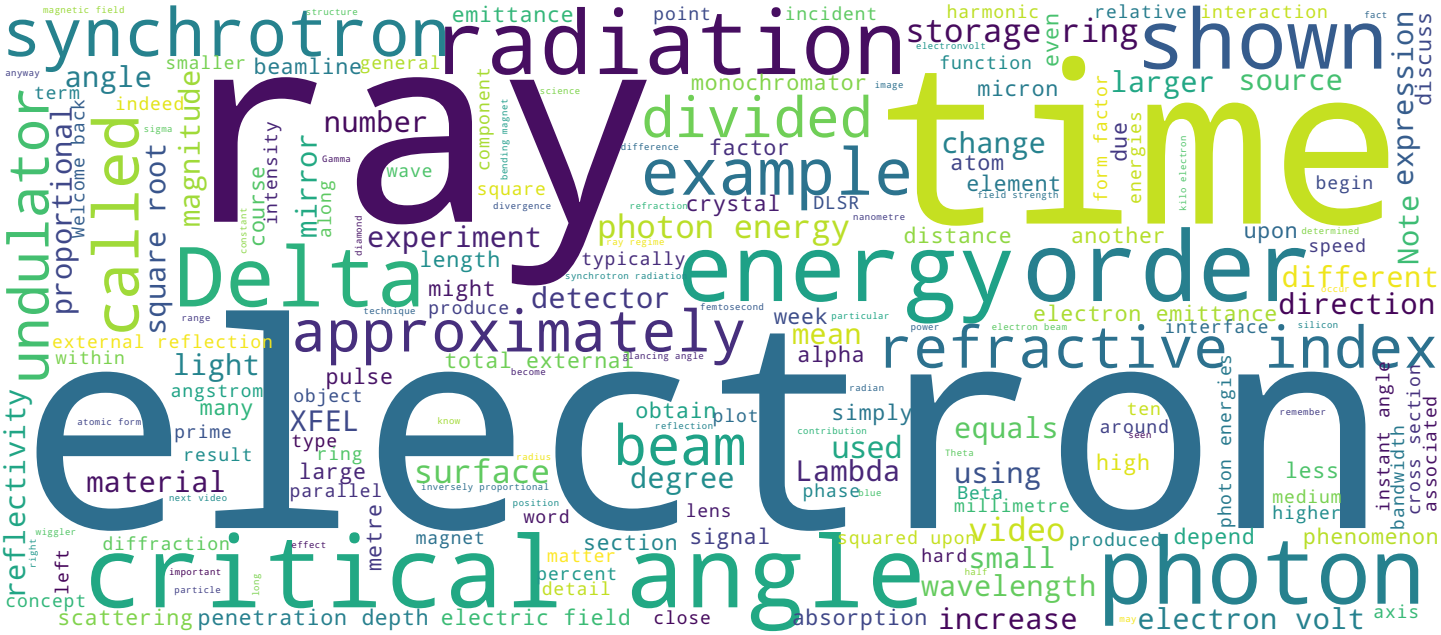

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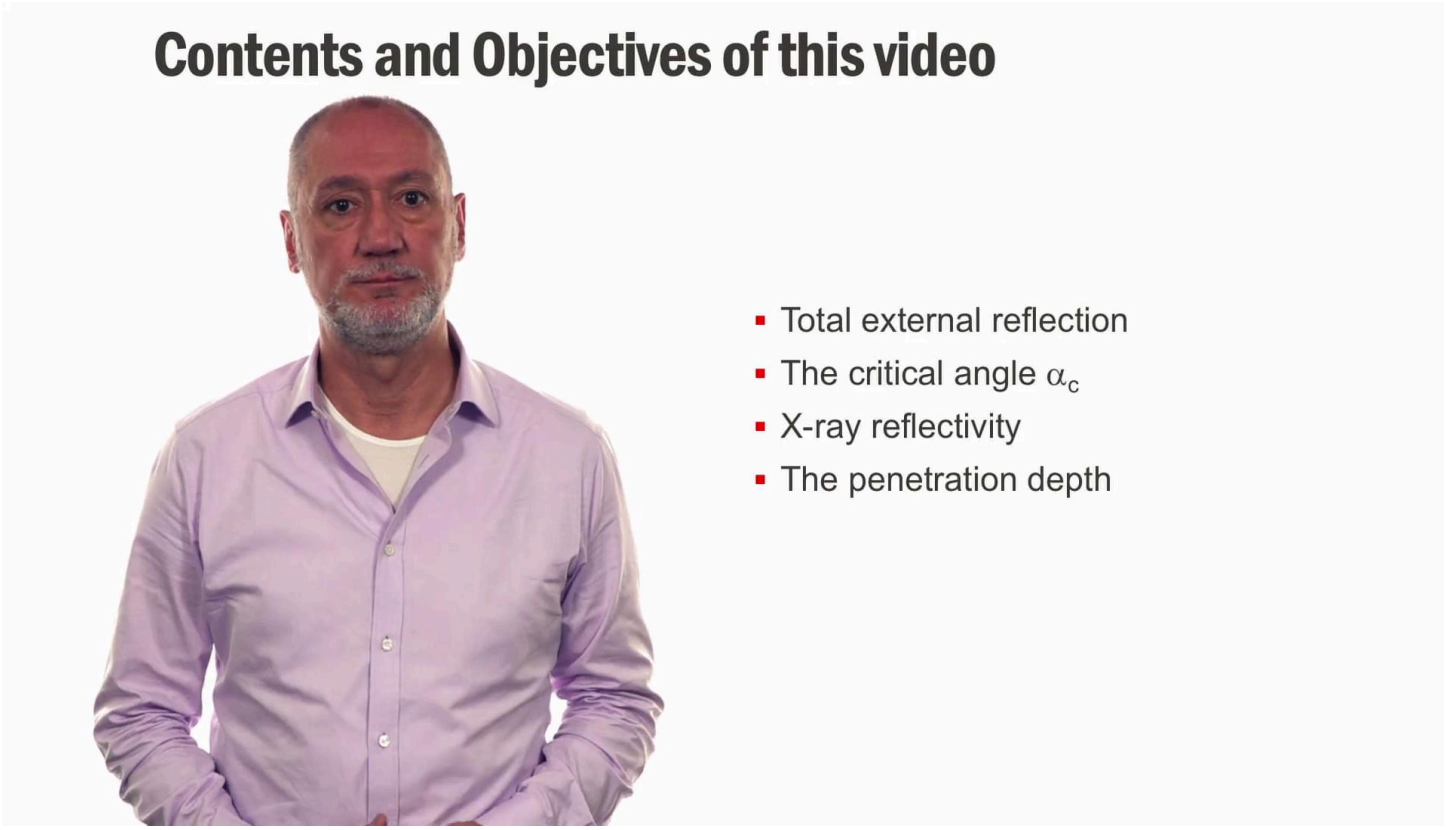
Video



Welcome back. In this video, we will discuss the concept of total external reflection of x-rays, including the critical angle which describes the maximum grazing angle at which total external reflection occurs, the degree of reflectivity and the penetration depth of x-rays above and below the critical angle.

Notes

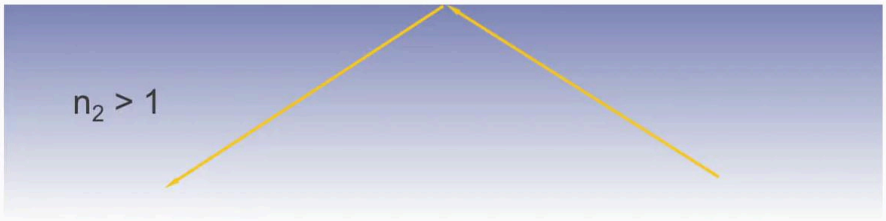
Summary

0m 05s

Total internal reflection in the visible

$$\frac{\cos \alpha_1}{\cos \alpha_2} = \frac{n_2}{n_1}$$

Vacuum, $n_1 = 1$



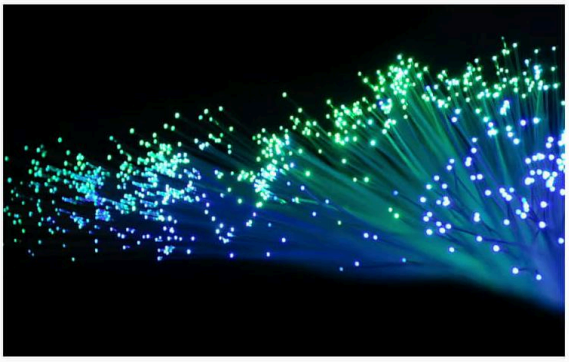
Total internal reflection is a phenomenon in the visible regime we have all seen once in a while. It occurs when light within one medium of refractive index N2 approaches the interface to another medium of lower refractive index N1 at a sufficiently glancing angle at or below the so-called critical angle. This phenomenon is a simple consequence of Snell's law. At a certain angle, the refracted ray becomes parallel to the surface, in other words, alpha one equals zero. As the angle of incidence decreases beyond this critical angle, the condition of refraction can no longer be satisfied. There is no refracted Ray and the ray incident on the interface is totally internally reflected.

Notes

Summary

0m 27s

Total internal reflection in the visible



$\alpha_{c,water} = 41^{o}$

Total internal reflection can be observed underwater, looking up to the surface at a glancing angle below that for which its cosine is larger than one upon the refractive index, that is one upon 1.33, leading to a critical angle of approximately 41 degrees. Total internal reflection is also ued in the infra-red for optical fibres at 1.55 microns. The infra-red radiation bounces off the interface between fused silica and a cladding material of lower refractive index. Using this technology, digital information can be transmitted at over 10 gigabytes per second over distances of around 100 kilometres before the signal requires re-amplification.

Notes

Summary

1m 24s

Total external reflection

"Total external reflection"

Now, we've already seen that in the x-ray regime, the refractive index of matter is smaller than one, but only by a very small amount. Now, a beam of x-rays entering a medium of refractive index N2 from, for example, vacuum, will be refracted to marginally shallower angles. Now following exactly the same arguments as before, there will be a minimum instant angle at which the refracted beam will be parallel to the surface. This instant angle is the critical angle for total external reflection and has a value determined by cosine alpha-C is equal to N2. For instant angles below alpha-C, the x-rays will therefore be totally externally reflected.

Notes

Summary

2m 15s

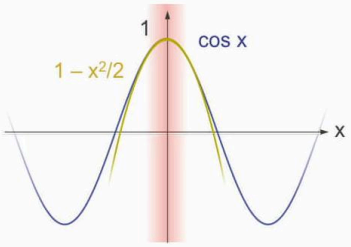
Magnitude of α_c

$$\cos\alpha_c = 1 - \delta$$

$$\cos\alpha_c \approx 1 - \alpha_c^2/2$$

$$\Rightarrow \delta \approx \alpha_c^2/2$$

$$\Rightarrow \boxed{\alpha_c \approx \sqrt{2\delta}}$$



e.g. $\delta = 5 \times 10^{-6}$
$\Rightarrow \alpha_c = 3.16 \times 10^{-3}$ rad = 0.18°

See also supplementary text "Trigonometric approximations"

What is the magnitude of alpha-C? Now we know the N2 is approximately equal to one minus Delta, where Delta is the refractive index decrement. We're going to ignore for the time being, the other term describing the refractive index, namely iBeta, which is anyway a size typically between one and three orders of magnitude smaller still than Delta. Because we know that Delta is small, cos alpha-C must be close to one, meaning the alpha-C is a very small angle. We can make the trigonometric approximation of cos X is equal to one minus X squared upon two for small values of X. For those of you unfamiliar with such trigonometric approximations, take a look at the supplementary document. But let's anyway look at this approximation graphically here. Shown in blue is cosine X where X is expressing radians. In gold, the function one minus X squared upon two is plotted. The small absolute values of X; the overlap between these two functions is remarkable, even out to a radian or thereabouts. That is sixty degrees. In any case, we can equate one minus alpha C squared upon two to one minus Delta, which means that to a high degree of accuracy, Delta equals alpha-C squared upon two and hence the critical angle for total external reflection is equal to the square root of two delta.

Notes

Summary

3m 08s

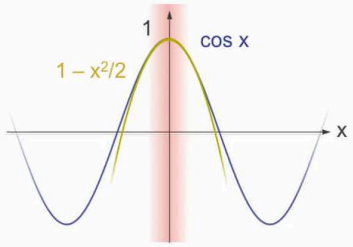
Magnitude of α_c

$$\cos\alpha_c = 1 - \delta$$

$$\cos\alpha_c \approx 1 - \alpha_c^2/2$$

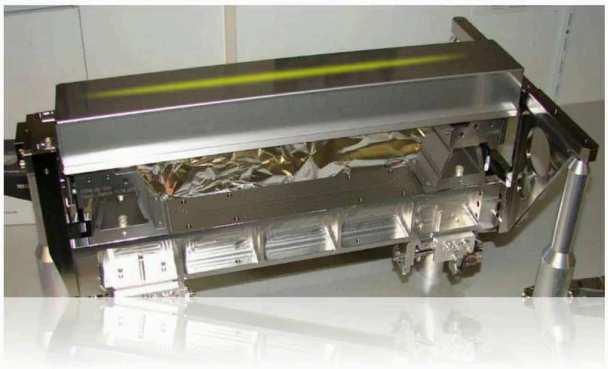
$$\Rightarrow \delta \approx \alpha_c^2/2$$

$$\Rightarrow \boxed{\alpha_c \approx \sqrt{2\delta}}$$



e.g. $\delta = 5 \times 10^{-6}$
$\Rightarrow \alpha_c = 3.16 \times 10^{-3}$ rad = 0.18°

See also supplementary text "Trigonometric approximations"



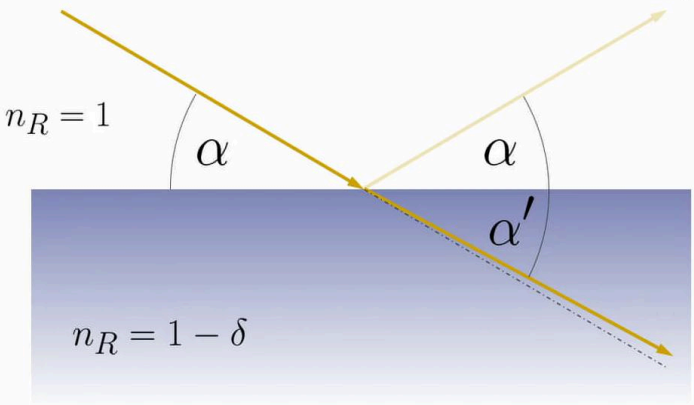
If we take a typical value of Delta of five times ten to the minus six, we obtain a critical angle of about three million radians or about 0.18 degrees. This phenomenon is used to construct x-ray mirrors. X-rays, incident on surfaces at angles lower than the critical angle will be totally externally reflected. Now, although synchrotron beams generally have low divergences, they may be of the order of a millimetre or more in height at a position where a mirror could be installed at a beamline. The footprint of such a beam, instant at a glancing angle shallower than the critical angle is therefore substantial and maybe tens of centimetres long. X-ray mirrors, therefore, tend to be large objects. The need to keep the smoothness of the mirror perfect down to an atomic scale therefore means that they're also extremely expensive.

Notes

Summary

4m 42s

X-ray reflectivity



$$R = r^2 = \left(\frac{\alpha - \alpha'}{\alpha + \alpha'}\right)^2$$

Snell's law

$$\frac{\cos\alpha}{\cos\alpha'} = 1 - \delta$$

$$R = \left[\frac{1 - (1 - 2\delta/\alpha^2)^{1/2}}{1 + (1 - 2\delta/\alpha^2)^{1/2}}\right]^2$$

See also derivation of R(α) in the supplementary material "X-ray reflectivity and penetration depths"

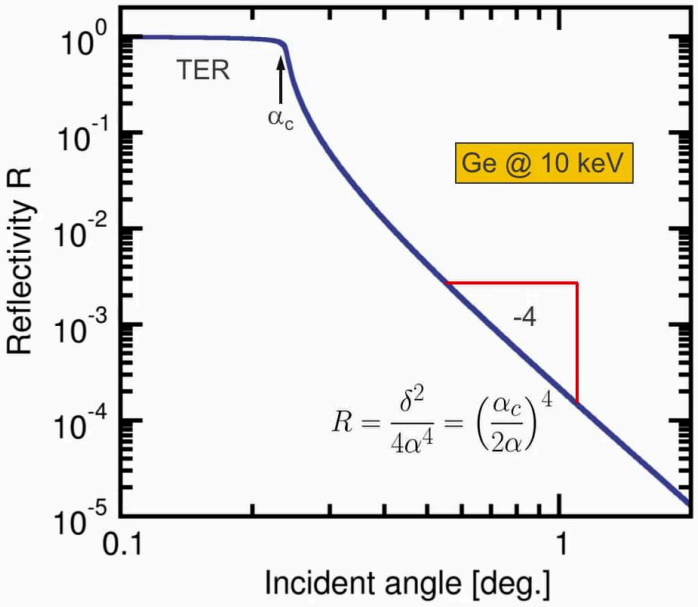
Let's take a closer look now to see how the reflectivity and transmission changes below at and above the critical angle. In general, the reflection intensity is equal to the square of the reflection amplitude, which depends on the incident and transmission angle, alpha and alpha prime – as given here. We can immediately see that at or below the critical angle, alpha prime is equal to zero and hence the reflectivity is 100 percent. Using Snell's law, we can convert this equation to want expressed in terms of Delta and Alpha instead of Alpha and Alpha Prime. Take a look at the supplementary document x-ray ray reflectivity and penetration tests for a derivation of this and the expressions that are about to follow.

Notes

Summary

5m 40s

X-ray reflectivity



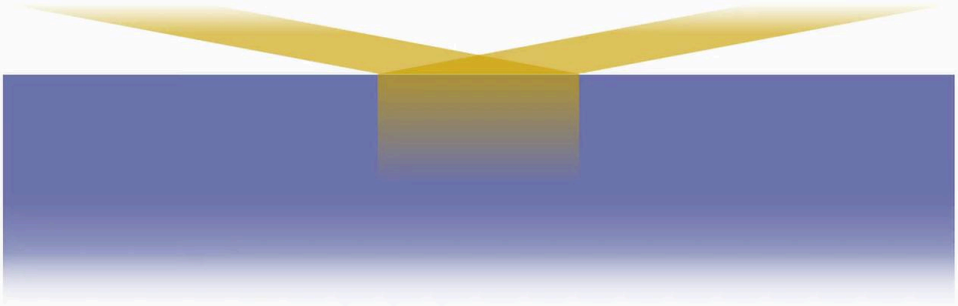
We see here a plot of the x-ray reflectivity of germanium for 10 kilo electron volt photons in this double logarithmic plot. The critical angle is approximately 0.24 degrees, below which, reflectivity is 100 percent. At angles substantially larger than alpha-C, the drop-off off in reflectivity goes as the inverse fourth power of the instant angle as attested to by the gradient of -4 in this double logarithmic plot. The derivation for this can be found in the supplementary material.

Notes

Summary

6m 33s

Penetration depth



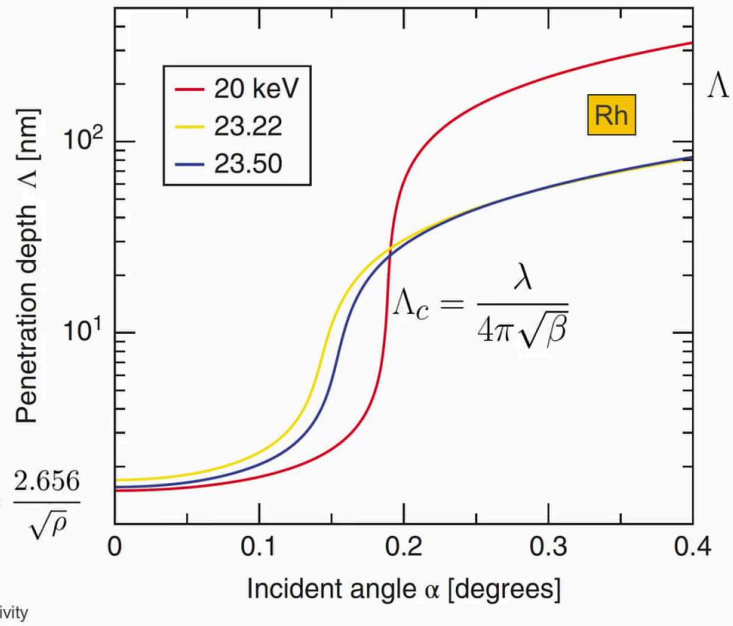
To finish this video, we need to also consider what's going on under the surface. Although below the critical angle we have stated that reflectivity is 100 percent, there must be some kind of interaction going on in order for a reflection to happen at all. This wave that penetrates below the surface is called the evanescent wave.

Notes

Summary

7m 12s

Penetration depth



$$\Lambda_0 = \frac{1}{4\sqrt{\pi r_0 \rho}} = \frac{2.656}{\sqrt{\rho}}$$

See also the supplementary material "X-ray reflectivity and penetration depths"

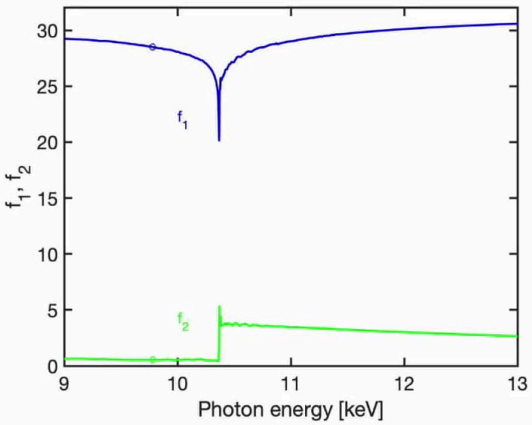
Below the critical angle, the x-rays rays penetrate down into the material only by a few angstroms, which makes it a sensitive probe for surface and interface properties. Here, the penetration depth lambda zero only depends on the inverse square root of the perceived electron density row, and in nanometres, is equal to 2.656 divided by the square root of row. At angles well above the critical angle, the penetration depth is proportional to the incident angle and x-ray wavelength and inversely proportional to the extinction coefficient beta. Exactly at the critical angle, the penetration depth is proportional to the x-ray wavelength and inversely proportional to the square root of beta.

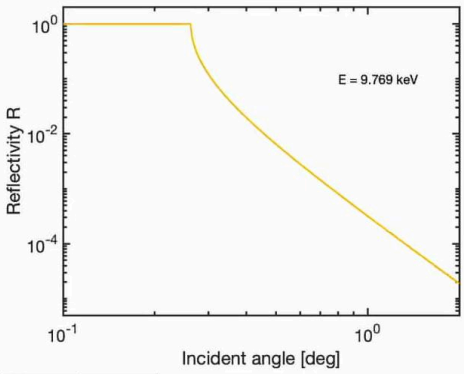
Notes

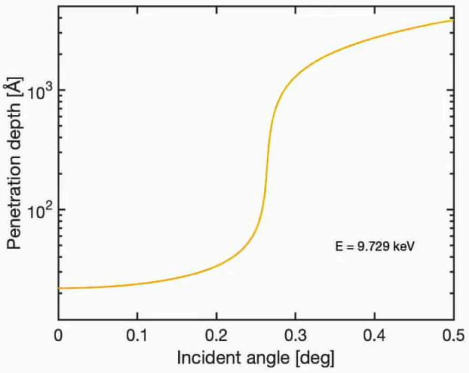
Summary

7m 35s

Reflectivity and penetration depth







Example:
Ga close to its K-edge

So we'll summarise now this video by seeing how the reflectivity and penetration depth of gallium varies close to its K absorption edge at 10.367 Kiloelectron volts. We track the changes in the atomic form factors, real and imaginary components, F1 and F2 with the small circles in the plot on the left. We slow things down close to the edge where things start to get interesting. Note in particular, how the penetration depth suddenly drops at the absorption edge due to sudden increase in Beta and F2. In the final video of this week, we will look more closely at photoabsorption and the processes that can occur subsequent to this and how the energy gained by the photoelectron is dissipated either through fluorescence or the ejection of a so-called [inaudible 00:09:23] electron plus the production of secondary electron cascades if the photoelectron has enough energy to completely leave its parent atom.

Notes

Summary

8m 28s